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Solution of Multi Objective Transportation Problem

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ABSTRACT
The transportation problem is one of the earliest applications of the linear programming problems. The basic transportation problem was originally developed by Hitchcock. Efficient methods of solution derived from the simplex algorithm were developed in 1947. The transportation problem can be modeled as a standard linear programming problem, which can then be solved by the simplex method. The objective of traditional transportation is to determine the optimal transportation pattern of a certain goods from supplier to demand customer so that the transportation cost become minimum and for this purpose we have different method for getting initial and optimal solution. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row minima, Column minima, Matrix minima, or the Vogel Approximation Method (VAM). To get an optimal solution for the transportation problem, we use the MODI method (Modified Distribution Method). In this paper we have developed an algorithm by fuzzy programming technique to solve multi objective transportation problem. We have also compared the result with raw maxima and EMV and show how the developed approach is more effective than other approaches.

KEYWORDS: *Multi- objective, Transportation, Fuzzy Programming, Cost, Time*

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1. INTRODUCTION

Two types of research work is done for transportation problem one is formulation of simple and multi objective transportation problem and second is developed a solution approach for simple and multi objective transportation problem. This chapter includes the work done on multi objective transportation problems as well as research objective. The transportation problem was formalized by the French Mathematician (Gaspard Monge, 1781). Major advances were made in the field during World War two by the Sovi- et/Russian mathematician and economist Leonid Vitaliyevich Kantorovich. Kantorovich is regarded as the founder of linear programming. Consequently, the problem as it is stated is sometimes known as the Monge–Kantorovich transportation problem.

In 2014, Sudipta Midya and Sankar Kumar Roy [1] solved single sink, fixed-charge, multiobjective, multi-index stochastic transportation problem by using fuzzy programming approach. A utility function approach to solve multi objective transportation problem was given by Gurupada Maity and Sankar Kumar Roy [2] in 2014. Then, in 2016 Gurupada Maity, Sankar Kumar Roy, and José Luis Verdegay [3] gave concept of cost reliability in the transportation cost and they considered supply and demand as uncertain variables. Sheema Sadia, Neha Gupta and Qazi M. Ali [4] presented their study on multi objective capacitated fractional transportation problem in 2016. They considered mixed linear constraints. Solution of multi objective transportation problem with non linear cost and multi choice demand was given by Gurupada Maity and Sankar Kumar [5] Roy in 2016. Mohammad Asim Nomani, Irfan Ali and A. Ahmed [6] presented algorithm of proposed method in 2017. In proposed method they have used weighted sum method based on goal programming. In 2017 only, Sankar Kumar Roy, Gurupada Maity, Gerhard Wilhelm Weber and Sirma Zeynep Alparslan Gök [7] solved multi objective transportation problem by using conic scalarization approach with interval goal. Then, by using utility approach with goals Sankar Kumar Roy, Gurupada Maity and Gerhard-Wilhelm Weber [8] solved multi objective two stage grey transportation problem in 2017. By inspired from Zimmermann's fuzzy programming and the neutrosophic set terminology recently in 2018, Rizk M. Rizk-Allah, Aboul Ella Hassanien and Mohamed Elhoseny [9] proposed a model under neutrosophic environment. In this model for each objective functions, they considered three membership functions namely, truth membership, indeterminacy membership and falsity membership. Srikant Gupta, Irfan Ali and Aquil Ahmed [10] presented their study on multi objective capaciated transportation problem with uncertain supply and demand. They formulated deterministic form of the problem by using solution procedure of multi choice and fuzzy numbers. Then they used goal programming approach to solve fractional objective function.

Many researchers have done tremendous work with this method, which is not mentioned all but some of the research is summarized over here. It is just a brief summary of fuzzy programming technique based optimization to provide comprehensive knowledge of fuzzy optimization and solutions. In real situation the

objective parameter are decided according to the requirement of decision maker. Many times he is unable to give such kind of information and to deal with these imprecision the parameter are formulated as fuzzy number, especially as triangular fuzzy numbers. Means, the objective function is fuzzified and leverage is provided to the decision maker to operate. Zadeh [11] first introduced the concept of fuzzy set theory. Then Zimmermann [12] first applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with several objective functions. He showed that solutions obtained by fuzzy linear programming are always efficient. Bit et al. [13] applied the fuzzy programming technique with linear membership function to solve the multi-objective transportation problem.

In this paper we have find the solution of multi objective transportation problems by fuzzy programming technique using linear membership function.

2. Multi objective Transportation Problem:

Let us consider that any company has m production centres, say $P_1, P_2, P_3, \dots \dots P_m$ and n warehouses or markets, say $W_1, W_2, W_3, \dots \dots W_n$. Let supply capacities of each production centers be $a_1, a_2, a_3, \dots \dots a_m$ respectively and demand levels of each destinations be $b_1, b_2, b_3, \dots \dots b_n$ respectively. The decision maker or manager of company wants to optimize r number of penalties $(z_r, where\ r = 1,2,3 \dots \dots K.)$ like minimize the transportation cost, maximize the profit, minimize the transportation time, minimize the risk, etc. Now, if C_{ij}^r be the cost associated with r^{th} objective to transport a unit product from i^{th} production center to j^{th} warehouse and X_{ij} be the unknown quantity to be transported from i^{th} production center to j^{th} warehouse. Then to solve this type of problem the multi objective transportation problem is defined as shown in model (1.2). [3]

Model: Multi objective transportation problem

$$\text{Minimize: } Z^r = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^r X_{ij},\ r = 1,2,3, \dots \dots \dots K;$$

$$\text{Subject to the constraints } \sum_{j=1}^{n} X_{ij} = a_i,$$
$$i = 1,2,3, \dots \dots \dots m;$$

$$\sum_{i=1}^{m} X_{ij} = b_j, \qquad j = 1,2,3, \dots \dots \dots n;$$

$X_{ij} \geq 0,\ \ \forall\, i, j.$

Where, $m =$ Number of sources;

$n =$ Number of destinations;

$a_i =$ Available supplies at i^{th} source;

$b_j =$ Demand level of j^{th} destination;

C_{ij}^r =Cost associated with r^{th} objective for transporting a unit of product from i^{th} source to j^{th} destination;

$X_{ij} =$ The quantity of product to be transported from i^{th} source to j^{th} destination.

To find optimal solution of any multi objective transportation problem, so many approaches are there like, goal programming, fuzzy approach, genetic algorithm, etc. In most of the solution approaches one general objective function is defined by considering each single objective function as the constraints. Solution given by general objective function may or may not give optimal solution to each objective function but this will give us compromise solution.

3. Fuzzy Programming Technique to Solve Multi-Objective Problems

Most of the entrepreneur now a day's do not have a aim of single objective but they wish to target multi objective i.e.` they not only try to minimize cost but try to minimize some recourse so that their business can grow in best of manner. In competitive world entrepreneur need to be aware of competition and should monopolized business. Their important objective could be to minimize risk using the same set of constraints. Such general multi objective linear programming problem can be defined as under [14,15]

$$\text{Minimize } z_k = \sum_{i=1}^{i=n} c_i^k x_i,\ k = 1,2,3,4\ldots\ldots,r$$

Subject to the constraints,

$$\sum_{i=1}^{n} a_i x_i (\leq,=,\geq) B_j,\ j = 1,2,3\ldots\ldots m\,,$$

$$x_i \geq 0.$$

In fuzzy programming technique following procedure applied to solve the multi objective optimization problem [12]:

The formulated multi objective linear programming problem first solve by using single objective function and derive optimal solution say $f_1(x_1, x_2, x_3 \ldots\ldots\ldots x_n)$ for first objective z_{11} and then obtain other objective value with the same solution say $z_{21}\ z_{31}\ z_{41} \ldots z_{k1}$. Procedure repeats same for $z_2 \ldots\ldots. z_r$ objectives.

Step 2: Corresponding to above data we can construct a pay off matrix which can give various alternate optimal value.

	Z_1	Z_2	………	Z_r	
$f_1(x_1, x_2, x_3 \ldots\ldots\ldots x_n)$	Z_{11}	Z_{21}	………	Z_{r1}	
$f_2(x_1, x_2, x_3 \ldots\ldots\ldots x_n)$	Z_{12}	Z_{22}	………	Z_{r2}	
……			………		
$f_n(x_1, x_2, x_3 \ldots\ldots\ldots x_n)$	Z_{1n}	Z_{2n}	………	Z_{rn}	

Table: 1- Pay-off matrix for MOLPP

Here,

z_{ki} : indicated optimal solution of 'k[th] objective using solution of 'i[th] objective, $k = 1,2,3,4\ldots\ldots,r$ and $i = 1,2,3\ldots\ldots n$.

Or

Find out the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function of the model

Now, by using pay-off matrix or positive ideal solution (PIS) and negative ideal solution (NIS) define a membership function $\mu(Z_k)$for the k^{th}objective function. Here two different membership function are utilized to find efficient solution of this multi-objective resource allocation problem and by using this membership function convert the MOLPP into the following model

Model -1:

Maximum λ,

Subject to the constraints

$$\lambda \le \mu(Z_k),$$

$$\sum_{i=1}^{m} a_i x_i (\le,=,\ge) B_j,\ j=1,2,3......n$$

$$x_i \ge 0,$$

When we utilize Fuzzy linear membership function [12] then model structure is as follows

Model- 2:

Maximum λ,

Subject to the constraints

$$z_k + \lambda(U_k - L_k) \le U_k,$$

$$\sum_{i=1}^{m} a_i x_i (\le,=,\ge) B_j,\ j=1,2,3......n$$

$$x_i \ge 0.$$

Solution of this model will give you an efficient solution

4. Algorithm to solve Multi-Objective Linear Programming Problem

Input: Parameters: $(Z_1, Z_2, ..., Z_k, n)$

Output: Solution of multi-objective programming problem

Solve multi-objective programming problem ($Z_k \downarrow, X \uparrow$)

begin

read: problem

while problem = multi-objective programming problem do

for k=1 to m do

enter matrix Z_k

end

-| determine pay-off matrix

Or

-| the positive ideal solution and negative ideal solution for each objective.

for k=1 to m do

$$z_{ij}^{\text{PIS}} = \min\left(z_i\right)^0$$

Under given constraints

end

for k=1 to m do

$$z_{ij}^{\text{NIS}} = \max\left(z_i\right)^0$$

Under given constraints

end

- find single objective optimization models under given constraints from multi-objective optimization models.

fork=1 to m do

max λ

Subject to the constraints:

$$\lambda \le \mu_{Z_{ij}}^{E}(x)$$

Under given constraints

End

|- find the solution SOPs using Lingo software.

5. Numerical Examples

This section considers several numerical examples of transportation problem and finds their solution by fuzzy programming technique

Numerical Illustration 1:

Transportation problem with some demand and supply are given below [21].

	D_1	D_2	D_3	D_4	Supply
S_1	1	2	7	7	8
S_2	1	9	3	4	19
S_3	8	9	4	6	17
Demand	11	3	14	16	

Table 2 showing objective function 1

	D1	D2	D3	D4	Supply
S_1	4	4	3	3	8
S_2	5	8	9	10	19
S_3	6	2	5	1	17
Demand	11	3	14	16	

Table 3 showing objective function 2

Mathematical Formulation of this problem can be written as

Min $Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$

Min $Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + x_{34}$

Subject to the constraints
$x_{11}+x_{12}+x_{13}+x_{14}=8$;
$x_{21}+x_{22}+x_{23}+x_{24}=19$;
$x_{31}+x_{32}+x_{33}+x_{34}=17$;

$x_{11}+x_{21}+x_{31}=11$;
$x_{12}+x_{22}+x_{32}=3$;
$x_{13}+x_{23}+x_{33}=14$;
$x_{14}+x_{24}+x_{34}=16$;

$x_{11}\ge 0$; $x_{12}\ge 0$; $x_{13}\ge 0$; $x_{14}\ge 0$;

$x_{21} \geq 0$; $x_{22} \geq 0$; $x_{23} \geq 0$; $x_{24} \geq 0$; $x_{31} \geq 0$; $x_{32} \geq 0$; $x_{33} \geq 0$; $x_{34} \geq 0$;

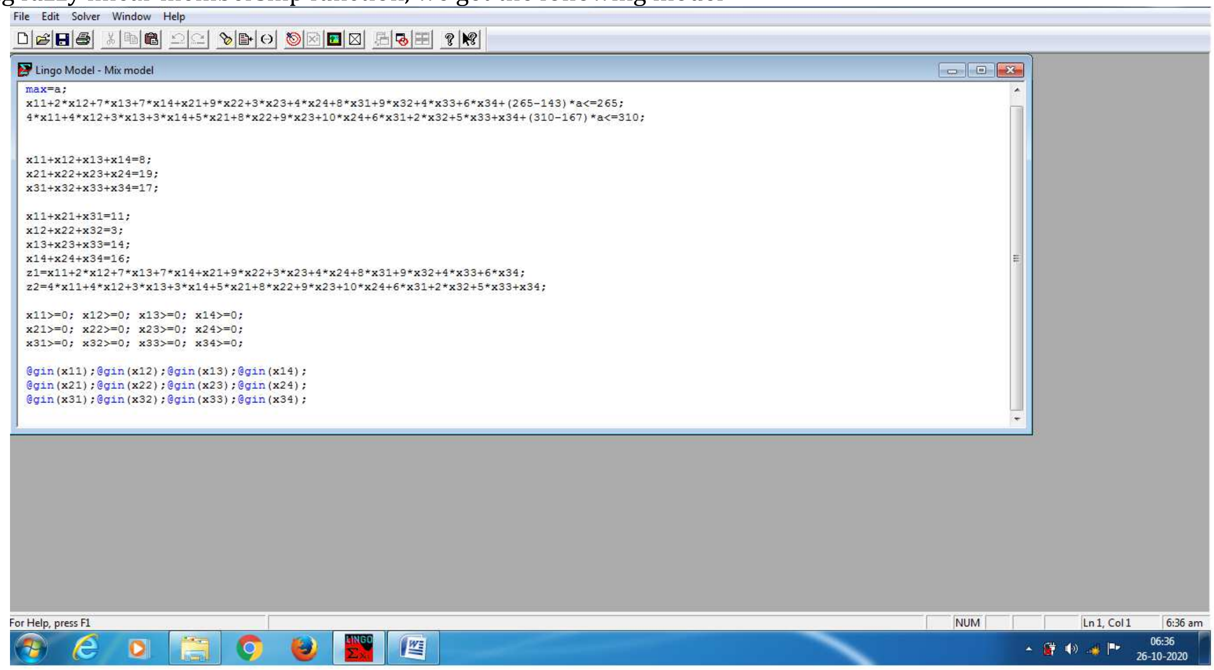
PIS and NIS value of first objective function is given by
PIS = 143, NIS = 265

PIS and NIS value of second objective function is given by
PIS = 167, NIS = 310

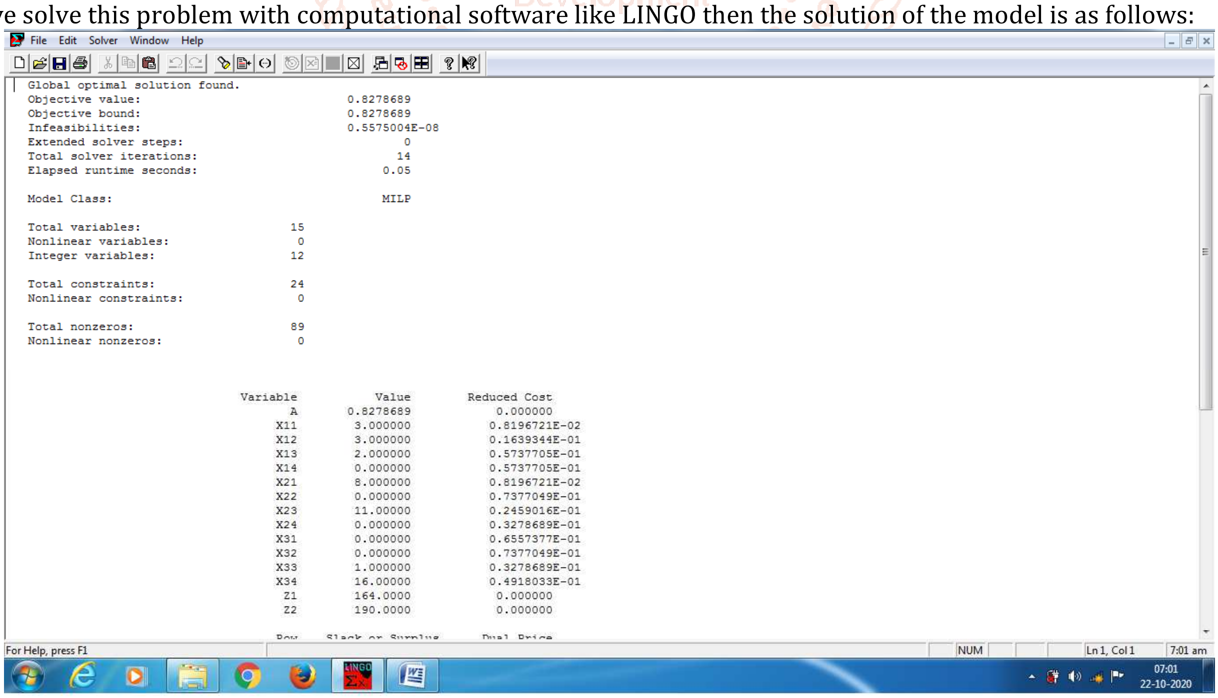
Hence,
$U_1=265$, $L_1=143$, $U_2= 310$, $L_2=167$

$U_1 - L_1 = 122$
$U_2 - L_2 = 143$

Applying fuzzy linear membership function, we get the following model



When we solve this problem with computational software like LINGO then the solution of the model is as follows:



The allocations are,
$X_{11} = 3.000000$ $X_{21} = 8.000000$ $X_{33} = 1.000000$
$X_{12} = 3.000000$ $X_{23} = 11.00000$ $X_{34} = 16.00000$
$X_{13} = 2.000000$

The values of objective functions are as follows: $Z_1 = 164.0000$, $Z_2 = 190.0000$

Using these allocations we have $Z_1=164$, $Z_2= 190$ with degree of satisfaction=0.8278689

The table given below shows the comparison of the given transportation problem with other approaches

Method	Numerical Example
Row maxima method [19]	Z_1= 172,Z_2=213.
EMV method [20]	Z_1=143, Z_2=265
fuzzy Multiobjective programming	Z_1= 164, Z_2= 190.

Numerical illustration -2:

A supplier, supply a product to different destination from different sources. The supplier has to take decision in this TP so that the transportation cost and transportation time should be minimum[21]. The data for the cost and time is as follows:

Destination→ sources↓	D1	D2	D3	Supply
S1	16	19	12	14
S2	22	13	19	16
S3	14	28	8	12
Demand	10	15	17	42

Table 4 Data for time

Destination→ sources↓	D_1	D_2	D_3	SUPPLY
S1	9	14	12	14
S2	16	10	14	16
S3	8	20	6	12
Demand	10	15	17	42

Table 5 Data for cost

The table given below shows the comparison of the given transportation problem with other approaches

Method	Objectives values
Row maxima method [19]	Z1= 518, Z2=374
EMV method [20]	Z1= 518, Z2=374
fuzzy Multi objective programming	Z1= 518, Z2=374

Numerical Illustration 3:

The data is collected by a person, who supplies product to different companies after taking it from different origins. There are four different suppliers named as A, B, C and D and four demand destinations namely E, F, G and H. How much amount of material is supplied from different origins to all other demand destinations so that total cost of transportation and product impairment is minimum [17].

Supplies: a1 = 21, a2 = 24, a3 = 18, a4 = 30.

Demands: b1 = 15, b2 = 22, b3 = 26, b4 = 30.

Table 6 Transportation cost for TP

	E	F	G	H	Supply
A	24	29	18	23	21
B	33	20	29	32	24
C	21	42	12	20	18
D	25	30	1	24	30
Demand	15	22	26	30	

Table 7 Product Impairment for TP

	E	F	G	H	Supply
A	24	29	18	23	21
B	33	20	29	32	24
C	21	42	12	20	18
D	25	30	1	24	30
Demand	15	22	26	30	

The table given below shows the comparison of the given transportation problem with other approaches

Objectives	Osuji et. al. (LMF) [17]	Osuji et. al. (HMF) [17]	Jignash G Patel GSDMT Method [21]	Fuzzy Programming Approach
Cost	1900	1898	1902	1902
Product impairment	1279	1286	1198	1198

Numerical Illustration 4:

Illustration 4: A company has four origins A, B, C and D with production capability of 5, 4, 2 and 9 units of manufactured goods, respectively. These units are to be transported to fivewarehouses E, F, G, H and I with necessity of 4, 4, 6, 2 and 4 units, respectively. The transportation cost, risk and product impairment between companies to warehouses are given below [18].

Table 8 Transportation cost for TP

	E	F	G	H	I	Supply
A	9	12	9	6	9	5
B	7	3	7	7	5	4
C	6	5	9	11	3	2
D	6	8	11	2	2	9
Demand	4	4	6	2	4	

Table 9 Transportation cost for TP

	E	F	G	H	I	Supply
A	2	9	8	1	4	5
B	1	9	9	5	2	4
C	8	1	8	4	5	2
D	2	8	6	9	8	9
Demand	4	4	6	2	4	

The table given below shows the comparison of the given transportation problem with other approaches

Objectives	Abo-Elnaga et. al. [18].	Jignash G Patel GSDMT Method [21]	Fuzzy Programming Approach
Cost	144	155	129
Risk	104	90	97
Product impairment	173	80	83

The comparison of four shows that the solution obtained by fuzzy programming technique gives better solution of multi objective transportation problem

Conclusion

This paper discussed a fuzzy programming technique for solution of multi objective transportation problem. With numerical illustrations, we conclude that fuzzy programming technique is a good alternative approach for find the solution of multi objective transportation problem

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